Math 85 midterm reprise -- open to all assistance -- answer in large exam book -- due on nov 14

Multiple Choice
Identify the choice that best completes the statement or answers the question. Show work or explain your choice, as appropriate.

1. Assume that $a > 0$, $b > 0$. The autonomous differential equation $\frac{dP}{dt} = P(a + bP)$ has a solution that is
   Select the correct answer.
   
   a. increasing everywhere
   b. decreasing everywhere
   c. increasing if $-a/b < P < 0$
   d. decreasing if $-a/b < P < 0$
   e. decreasing if $P < -a/b$

Multiple Response
Identify one or more choices that best complete the statement or answer the question.

2. Which of the following sets of functions are linearly independent on $(0, \infty)$? Select all that apply. Give justification for choosing/omitting each one.
   
   a. $\{1, \sin^2 x, \cos^2 x\}$
   b. $\{1, x + 3, 2x\}$
   c. $\{\sqrt{x}, x, x^2\}$
   d. $\{1, \tan^2 x, \sec^2 x\}$
   e. $\{1/x, x, \ln x\}$
Short Answer
Be sure to include justification for your answer (i.e., show work or give explanation, as appropriate).

3. A large mixing tank initially contains 1000 gallons of water in which 40 pounds of salt have been dissolved. Another brine solution is pumped into the tank at the rate of 5 gallons per minute, and the resulting mixture is pumped out at the same rate. The concentration of the incoming brine solution is 3 pounds of salt per gallon. If \( A(t) \) represents the amount of salt in the tank at time \( t \), the correct differential equation for \( A \) is _____.

4. The solution of the initial value problem
\[
y' = 2y + x, \quad y(-1) = 1/2
\]
is
\[
y = -x/2 - 1/4 + ce^{2x}, \text{ where } c = _____.
\]

5. The temperature of a cup of coffee obeys Newton's law of cooling. The initial temperature of the coffee is 140°F and one minute later, it is 125°F. The ambient temperature of the room is 65°F. If \( T(t) \) represents the temperature of the coffee at time \( t \), the correct differential equation for the temperature is _____.

6. Solve the problem \( y' = (x + 1)y, \quad y(0) = 1 \) numerically for \( y(0.2) \) using \( h = 0.1 \).

7. The differential equation \( (x + 2y)dx + ydy = 0 \) can be solved using the substitution _____.

8. The half-life of plutonium 239 is 24,200 years. Assume that the decay rate is proportional to the amount. An initial amount of 3 grams of radium would decay to 2 grams in approximately _____.

9. The population of a certain town doubles in 14 years. How long will it take for the population to triple? Assume that the rate of increase of the population is proportional to the population.

10. A ball is thrown upward from the top of a 200 foot tall building with a velocity of 40 feet per second. Take the positive direction upward and the origin of the coordinate system at ground level. What is the initial value problem for the position, \( x(t) \), of the ball at time \( t \)?

11. A particular solution of the differential equation \( y'' + 3y' + 4y = 8x + 2 \) is _____.

12. After substituting, the auxiliary equation for the differential equation \( x^2y'' + 5xy' + 4y = 6 \) is _____.

13. The solution of the system of differential equations
\[
\begin{align*}
\frac{dx}{dt} &= x + 2y \\
\frac{dy}{dt} &= 4x + 3y
\end{align*}
\]
is _____.