Using **Mathematical Induction**
to prove the "Sum" formulas for the first n terms of a given sequence.

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**Step 1**
Show that $S_n$ is true for $n = 1$ by showing that $S_1 = a_1$.

**Step 2**
Show that $S_k$ implies $S_{k+1}$.

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**Example**
Prove that $S_n = n(2n + 1)$ for $a_n = 4n - 1$.

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**Note 1:** Replacing $n$ in $a_n$ above with 1, 2, 3, 4, ... produces the sequence 3, 7, 11, 15, ..., $(4n-1)$, ... The formula $S_n$ is "claimed" to be the shortcut for finding the sum of the first "n" of these terms. It is your job to prove that this formula will always work no matter what "n" is.

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**Step 1**

You Write This

\[ S_1 = 1(2(1) + 1) = 1(3) = 3 \]

Discussion

You have shown that $S_1 = a_1$.

Step 1 is finished.

**Step 2**

You Write This

\[ S_{k+1} = S_k + a_{k+1} \]

Discussion

Copy this identity to start.

\[ = k(2k + 1) + 4(k + 1) - 1 \]

Substituted for both $S_k$ and $a_{k+1}$.

\[ = 2k^2 + k + 4k + 4 - 1 \]

Since factoring is not yet available, simplify.

\[ = 2k^2 + 5k + 3 \]

Simplified more.

\[ = (k + 1)(2k + 3) \]

Factored the trinomial.

\[ = (k + 1)(2(k + 1) + 1) \]

Rewritten so that it looks exactly like what $S_n$ would look like if $n$ were replaced by $k + 1$.

Therefore, we have proved that $S_n = n(2n + 1)$ will yield the correct sum for any number of terms of the sequence $a_n = 4n - 1$.

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**Note 2:** Some instructors want you to start Step 2 by showing the "Inductive Hypothesis". This is simply the formula $S_n$ where each $n$ is replaced by $k$. For the example problem you would write $S_k = k(2k + 1)$. You are allowed to assume that this statement is true. In addition, it's worth mentioning that $k$, (like $n$) is a natural number.

**Note 3:** Also in step 2, you may find it easier if you jot down (off to the side) what your goal is. You do this by copying $S_n$ but replacing each $n$ with $k + 1$, using parenthesis where necessary.