Mathematical Induction
for proving inequalities

For convenience, we call the statement we're trying to prove $P_n$. ($P_n$ is a statement whose variable is $n$.)

Step 1  Show that $P_1$ is true. ($P_1$ is $P_n$ except $n$ is replaced by 1.)

Step 2  Show that $P_k$ implies $P_{k+1}$. ($P_k$ is $P_n$ except $n$ is replaced by $k$, etc.)

Example  Prove that $2^n \geq n + 1$ for all natural numbers $n$.  

Step 1  

You Write This

Discussion
don't write

$2^1 \geq 1 + 1$
$2 \geq 2 \checkmark$

We replaced $n$ with 1 in the statement we're trying to prove. Since it results in a true statement, we can proceed to Step 2.

Step 2  

You Write This

Discussion
don't write

Assume $P_n$ is true for natural number $k$, i.e.,

$2^k \geq k + 1$

Start with $P_k$. This is your inductive hypothesis. You are allowed to assume it's true. (It's just the original statement with $k$ in place of $n$.)

$2 \cdot 2^k \geq 2 \cdot (k + 1)$

Since our goal is to show that $P_k$ implies $P_{k+1}$, where $P_{k+1}$ is the statement $2^{k+1} \geq (k + 1) + 1$, we first want to get $2^{k+1}$ on the left hand side. We do this by multiplying both sides by 2.

$2^{k+1} \geq 2k + 2$

We simplified on both sides. Now, the left hand side is where we want it, it's time to work on the right hand side.

$2^{k+1} \geq (k + 1) + 1 + k$

The right hand side is good now except it has an extra $k$ at the end. As it turns out, this is not that big of a problem since $k$ is known to be greater than zero. (see above, $k$ is a natural number)

$2^{k+1} \geq (k + 1) + 1 + k \geq (k + 1) + 1$

This is true since $k \geq 0$.

$2^{k+1} \geq (k + 1) + 1$

(Done) This is true by the transitive property for inequalities which says if $a \geq b$, and $b \geq c$, then $a \geq c$. 

$2^{k+1} \geq (k + 1) + 1$