Finding Least Common Denominators

Least Common Denominators, in essence, are Least Common Multiples. This handout will start by examining the process of finding Least Common Multiples.

**DEFINITION:** The **Natural Numbers** (or counting numbers) are 1, 2, 3, 4, 5, ... .

**DEFINITION:** The **Multiples** of a number are the products obtained when you multiply that number by the Natural Numbers.

**Example 1:** List the first twelve multiples of: A) 6    B) 8

**Answer:** A) 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72

B) 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96

Please note that the **multiples** of a number are not the **factors** of that number. For example, the factors of the number 6 are 1, 2, 3, and 6, which are the numbers that divide into 6 (with a remainder of zero). The multiples of 6 (above) are the numbers which are divisible by 6 (giving a remainder of zero).

**Example 2:** Give three multiples common to both 6 and 8.

**Answer:** Looking at the lists above, we see that 6 and 8 have 24, 48, and 72 as common multiples. (There are many more. In fact, 6 and 8 have an infinite number of common multiples!)

**Example 3:** What is the Least Common Multiple (LCM) of 6 and 8?

**Answer:** The least (smallest) of the all of the common multiples of 6 and 8 is 24.

**FINDING LEAST COMMON MULTIPLES (LCMs)**

In the example above, we made lists of multiples and found our LCM in the lists. But in general, this method is inefficient and cumbersome, especially when working with larger numbers. Aside from some special cases (to be discussed later in this handout), the recommended process is as follows.

### Finding Least Common Multiples

1. **Step 1:** Find the prime factorization of all numbers using exponents.
2. **Step 2:** Multiply together the highest power of each different prime factor.

**Example 4:** Find the LCM of 6 and 8 (again) using the process just described.

Using factor trees and circling the prime numbers:

- 6: \(2 \times 3\)
- 8: \(2 \times 4\) or \(2 \times 2 \times 2\)

Now, writing the prime factorizations using exponents gives:

- 6: \(2^1 \times 3^1\)
- 8: \(2^3\)

The highest power of the twos is \(2^3\), and the highest power of the threes is \(3^1\).

This makes the LCM of 6 and 8 to be \(2^3 \times 3^1 = 8 \times 3 = 24\). 

Also be aware that the numbers 6 and 8 will be represented (or contained) in the final result, 24: \(2 \times 2 \times 2 \times 3 \div 6\).
Example 5: Find the LCM of 12 and 18

From the factor trees (right), we have:

\[ 12 = 2^2 \times 3^1 \]
\[ 18 = 2^1 \times 3^2 \]

The highest power of the twos is \(2^2\), and the highest power of the threes is \(3^2\), so the LCM of 12 and 18 is \(2^2 \times 3^2 = 36\).

**DEFINITION:** The **Least Common Denominator (LCD)** of a group of fractions is the Least Common Multiple (LCM) of the denominators.

Example 6: Find the Least Common Denominator for:

a) \( \frac{5}{12} \) and \( \frac{9}{14} \)

b) \( \frac{1}{6} \), \( \frac{9}{10} \), and \( \frac{7}{18} \)

(The process for finding LCDs is the same as for LCMs)

a) Since \( 12 = 2^2 \times 3^1 \), and \( 14 = 2^1 \times 7^1 \),

The LCD of \( \frac{5}{12} \) and \( \frac{9}{14} \) is \(2^2 \times 3^1 \times 7^1 = 84\)

b) \( 6 = 2^1 \times 3^1 \), \( 10 = 2^1 \times 5^1 \), \( 18 = 2^1 \times 3^2 \)

The LCD of \( \frac{1}{6} \), \( \frac{9}{10} \), and \( \frac{7}{18} \) is \(2^1 \times 3^2 \times 5^1 = 90\)

A Few Special Cases

I. If the biggest denominator is a multiple of the smaller ones, the biggest one will be the LCD.

II. If the denominators have no common factors (not counting 1), multiply the denominators to get your LCD.

III. Consider the multiples of the biggest denominator (in order). When you land on one that is a multiple of the smaller one(s), that will be the LCD.

Example 7: Find the Least Common Denominator for:

a) \( \frac{1}{4} \), \( \frac{3}{8} \), and \( \frac{7}{16} \)

Answer: Since the big denominator (16) is a multiple of the smaller ones, 4 and 8, the big denominator, 16, is it. (I)

Notice how this makes sense as both 4 and 8 are factors of the number 16.

\[ 16 = 2^4 = \frac{8}{4} \]

Since there is no "overlap" in the prime factors of these two numbers, 6 = 2 \times 3, and 7 = 7, just multiply them all together. Or, just multiply the denominators.

b) \( \frac{1}{6} \) and \( \frac{4}{7} \)

Answer: Since the denominators, 6 and 7, have no common factors, multiply them. \(6 \times 7 = 42\). (II)

c) \( \frac{5}{6} \) and \( \frac{3}{10} \)

Answer: The multiples of the big denominator, 10, are 10, 20, 30, … . Ask yourself, is 10 a multiple of 6? No. Is 20 of multiple of 6? No. Is 30 a multiple of 6? Yes! 30 is the LCD (III.)

d) \( \frac{5}{16} \) and \( \frac{7}{10} \)

Answer: While trying Special Case III is a possibility, here it's a little cumbersome. So, back to the original method we go. \( 16 = 2^4 \), and \( 10 = 2^1 \times 5^1 \), so the LCD is \(2^4 \times 5^1 = 16 \times 5 = 80\). 😊