Square Root Basics

**Definition:** A number “a” is a square root of another number “b”, if \( a^2 = b \).
For example: 5 is a square root of 25, since \( 5^2 = 25 \)

**Question:** What are the square roots of 9?

**Answer:** +3 or – 3, since \((+3)^2 = 9\) and \((-3)^2 = 9\).

**Question:** Then, what is \( \sqrt{9} \)?

**Answer:** The symbol \( \sqrt{\quad} \) is used to show the “principal” (or positive) square root of 9; it’s just +3.
Note: \( -\sqrt{9} = -3 \)

**Question:** How about \( \sqrt{7} \)?

**Answer:** While \( \sqrt{7} \) is not a whole number, it is a very precise, unique, (irrational) number. From the definition above, it is the number whose square is 7. In other words, \((\sqrt{7})^2 = 7\). Or, \( \sqrt{7} \cdot \sqrt{7} = 7\).
Note: Whenever an approximation is sufficient, a calculator can give that to you. \( \sqrt{7} \approx 2.645751311 \)

### Properties of Square Roots

\[
\sqrt{a} \cdot \sqrt{b} = \sqrt{ab} \\
\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \\
\text{If } a \text{ is negative, then } \sqrt{a} \text{ is not a real number.}
\]

\( \sqrt{a^2} = |a| \) (Because a principal square root is always non-negative, but a itself could be negative.)

**Numerical examples of definitions and properties:**

\[
\begin{align*}
\sqrt{64} &= 8 \\
-\sqrt{100} &= -10 \\
\sqrt{5} \cdot \sqrt{7} &= \sqrt{35} \\
\sqrt{\frac{10}{21}} &= \frac{\sqrt{10}}{\sqrt{21}}
\end{align*}
\]

\[
\sqrt{(-15)^2} = |-15| = 15 \\
\sqrt{-16} \text{ is not a real number}
\]

**Definitions:**

* The **square root sign** (or **radical sign**) is the symbol \( \sqrt{\quad} \)
* The **radicand** is the number or expression on the inside of the radical sign.

* On the inside of the little “v” part of the radical sign, you could put a number, like this: \( 3 \sqrt{\quad} \). This little number is called the **index**. If the index is not written, it is assumed to be a 2. An index of 2 tells you that this is a square root.

* All together, we call this kind of expression a **radical**, but more specifically, a square root. (Remember, it’s a principal square root if there’s no sign in front; it’s a negative square root if there’s a minus sign in front!)

**For example:** In the principal square root, \( \sqrt{3ab} \), 3ab is called the radicand, the index is 2, and the whole thing is called a radical. As a principal square root, its overall value will not be negative.