Conic Sections

Determining Which Conic Section You Have

Put (or just imagine putting) your equation in the form: \( Ax^2 + Cy^2 + Dx + Ey + F = 0 \)
Then, all you will need to do is compare A & C as follows:

- If \( A = C \) \( \rightarrow \) CIRCLE
- If \( A \) & \( C \) have opposite signs \( \rightarrow \) HYPERBOLA
- If \( A \) & \( C \) have same sign and \( A \neq C \) \( \rightarrow \) ELLIPSE
- If \( A \) or \( C = 0 \) (not both) \( \rightarrow \) PARABOLA

Note 1: If \( A = 0 \) and \( C = 0 \), you'll have a linear equation; the graph would be a line.

Note 2: A "trick question" is rare, but possible. A given equation could be a contradiction (no graph), or just have one solution, a single point. For example, \( 3x^2 + 4y^2 = -5 \) (positive + positive = negative), no solutions \( \rightarrow \) no graph. Or \( 3x^2 + 4y^2 = 0 \), one solution, \((0,0)\), graph is a single point.

Circles

\[(x - h)^2 + (y - k)^2 = r^2 \] Center at \((h, k)\) and a radius \(r\).

Parabolas

\[(x - h)^2 = 4p(y - k) \] \(\text{Vertex: } (h, k)\) Axix of Symmetry: \(x = h\) The focus lies on the axis of symmetry, \(p\) units from the vertex. The parabola will open up when \(p\) is positive, and down when \(p\) is negative. The directrix is a line perpendicular to the axis of symmetry, intersecting the axis of symmetry at a point \(|p|\) units behind the parabola.

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Ellipses

\[\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \ (a^2 \neq b^2)\]. The center of the ellipse will be at \((h, k)\). The endpoints of the major axis (called vertices) will be \(a\) or \(b\) units from the center, whichever is larger. The endpoints of the minor axis will be \(a\) or \(b\) units from the center, whichever is smaller. If \(a^2 > b^2\), the major axis will be horizontal. If \(b^2 > a^2\), the major axis will be vertical. There will be two foci that lie on the major axis, \(\pm c\) units from the center, where \(c^2 = |a^2 - b^2|\).

Hyperbolas

\[\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \] Center at \((h, k)\). Opens right and left with vertices \(\pm a\) units from the center.

\[\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1 \] Center at \((h, k)\). Opens up and down with vertices \(\pm b\) units from the center.

The line that connects the two vertices is called the transverse axis. There will be two foci on the transverse axis, \(\pm c\) units from the center, where \(c^2 = a^2 + b^2\). There will be two asymptotes that intersect at the center. Asymptote equations: \(y - k = \pm \frac{b}{a} (x - h)\)