The Gauss-Jordan Method for Solving Systems of Linear Equations

1 of <u>many</u> possible methods

2 Equations, 2 Variables

Start $\begin{bmatrix} * & * & * \\ * & * & * \end{bmatrix}$ **Copy the coefficients and constant terms in the same pattern that you see them.**

First $\begin{bmatrix} 1 & * & * \\ * & * & * \end{bmatrix}$ **Get a "1" in the top left position.**

2nd $\begin{bmatrix} 1 & * & * \\ 0 & * & * \end{bmatrix}$ **Get a "0" in the column of the "1".**

3rd $\begin{bmatrix} 1 & * & * \\ 0 & 1 & * \end{bmatrix}$ **Get a "1" in the 2nd row, 2nd column.**

Last $\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \end{bmatrix}$ **Get a "0" in the column of that "1".**

Answer: { (*a*, *b*) }

Note: These systems can be solved using a number of <u>different</u> approaches. However, most students are better off sticking to a single method, maybe this one.

3 Equations, 3 Variables

Start $\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$ **Copy the coefficients and constant terms in the same pattern that you see them.**

First $\begin{bmatrix} 1 & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$ **Get a "1" in the top left position.**

2nd $\begin{bmatrix} 1 & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{bmatrix}$ **Get zeros in the column of the "1".**

3rd $\begin{bmatrix} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & * & * & * \end{bmatrix}$ **Get a "1" in the 2nd row, 2nd column.**

4th $\begin{bmatrix} 1 & 0 & * & * \\ 0 & 1 & * & * \\ 0 & 0 & * & * \end{bmatrix}$ **Get zeros in the column of that "1".**

5th $\begin{bmatrix} 1 & 0 & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \end{bmatrix}$ **Get a "1" in the 3rd row, 3rd column.**

Last $\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{bmatrix}$ **Get zeros in the column of that "1".**

Answer: { (*a*, *b*, *c*) }

Summary: Get the 1 first, and then use that 1 to help get your zeros. **See <u>next page</u> for an explanation on how to do that.**

Are you familiar with the tools in your toolbox?

1. **Any two rows of the matrix can be interchanged (/switched/swapped).**
2. **Any row of the matrix can be multiplied by any (nonzero) number.**
3. **Any row of the matrix can be replaced by the sum of a (nonzero) multiple of another row added to a (nonzero) multiple of the row you're replacing.**

✳**Getting Zeros**: **Multiply the row with the "1" in it by the opposite of the number where you want the zero; then add to that the row you are replacing. That's it!**

Example: For $\begin{bmatrix} 1 & -3 & 8 \\ 2 & 5 & -6 \end{bmatrix}$**, replace Row 2 with** $(-2)R_1 + R_2$**. You'll get** $\begin{bmatrix} 1 & -3 & 8 \\ 0 & 11 & -22 \end{bmatrix}$**.**

✳**Getting One's**:

✳**In the first row**

A. **If there is a 1 in the first position of another row, just "swap" rows.**
B. **Consider the multiples of the first elements of each row and find 2 that are different by 1. Then replace row 1 with that "sum".**

Example: For $\begin{bmatrix} 3 & -2 & 5 \\ 7 & 1 & -2 \end{bmatrix}$**, replace Row 1 with** $R_2 + (-2)R_1$**. You'll get** $\begin{bmatrix} 1 & 5 & -12 \\ 7 & 1 & -2 \end{bmatrix}$**.**

or for $\begin{bmatrix} 5 & -3 & 7 \\ 7 & 2 & -1 \end{bmatrix}$**, replace Row 1 with** $(-2)R_2 + 3R_1$**. You'll get** $\begin{bmatrix} 1 & -13 & 23 \\ 7 & 2 & -1 \end{bmatrix}$**.**

C. **(My last resort)... Multiply row 1 by the reciprocal of the first element. This will probably bring fractions into the matrix, then, have fun working with those fractions.**

✳**In the bottom row**

This is pretty easy. Just multiply that bottom row by the reciprocal of the number where you want the 1.

✳**In the middle row** **(of a 3 equation, 3unknown system)**

A. **If the 3rd row has a 1 in the 2nd column, just swap rows 2 and 3.**
B. **Like step B (above), consider the multiples of the first elements of rows 2 and 3, and find a couple that are different by 1. Then replace row 2 with that "sum".**

Example: For $\begin{bmatrix} 1 & -6 & 8 & 7 \\ 0 & 4 & -2 & 5 \\ 0 & 9 & 3 & -1 \end{bmatrix}$**, replace row 2 with** $R_3 + (-2)R_2$**, to get** $\begin{bmatrix} 1 & -6 & 8 & 7 \\ 0 & 1 & 7 & -11 \\ 0 & 9 & 3 & -1 \end{bmatrix}$**.**

C. **Multiply row 2 by the reciprocal of the term where you want the 1. Again, this likely will bring fractions into the problem. I recommend using a standard elimination technique, get a zero in row 2, column 3, first.**

Example: For $\begin{bmatrix} 1 & 3 & -2 & 4 \\ 0 & 4 & 3 & 14 \\ 0 & 8 & 7 & 26 \end{bmatrix}$**, replace row 2 with $3R_3 + (-7)R_2$, to get**

$$\begin{bmatrix} 1 & 3 & -2 & 4 \\ 0 & -4 & 0 & -20 \\ 0 & 8 & 7 & 26 \end{bmatrix}.$$

<u>Then</u>, multiply row 2 by the reciprocal of that 2nd element! Good luck!