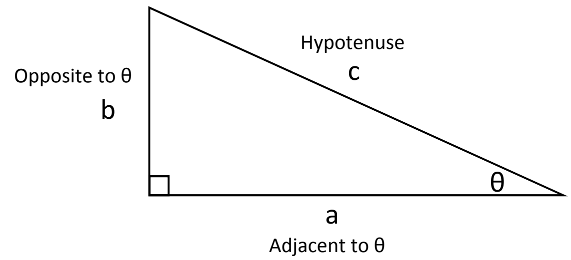
TRIGONOMETRIC IDENTITIES

For an Acute Angle in a right triangle:

$$\sin\theta = \frac{b}{c} = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \cos\theta = \frac{a}{c} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{b}{a} = \frac{\text{Opposite}}{\text{Adjacent}} \qquad \csc\theta = \frac{c}{b} = \frac{\text{Hypotenuse}}{\text{Opposite}}$$

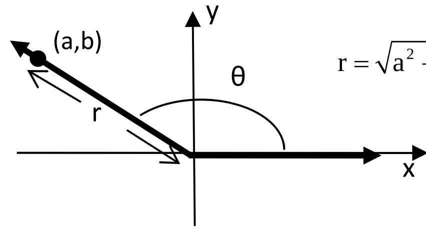
$$\sec\theta = \frac{c}{a} = \frac{\text{Hypotenuse}}{\text{Adjacent}} \qquad \cot\theta = \frac{a}{b} = \frac{\text{Adjacent}}{\text{Opposite}}$$



Of a General Angle:

$$\sin\theta = \frac{b}{r} \qquad \cos\theta = \frac{a}{r} \qquad \tan\theta = \frac{b}{a}, a \neq 0$$

$$\csc\theta = \frac{r}{b}, b \neq 0 \qquad \sec\theta = \frac{r}{a}, a \neq 0 \qquad \cot\theta = \frac{a}{b}, b \neq 0$$



$$r = \sqrt{a^2 + b^2}$$

Fundamental Identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

Even-Odd Identities:

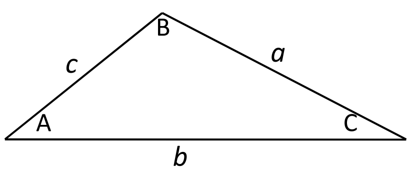
$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta \qquad \cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta \qquad \tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Sum and Difference Formulas:

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta \qquad \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \qquad \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

Solving Triangles



Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Half-Angle Formulas:

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

Double Angle Formulas:

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta \qquad \cos(2\theta) = 2\cos^2\theta - 1 \qquad \cos(2\theta) = 1 - 2\sin^2\theta$$

$$\sin(2\theta) = 2\sin\theta\cos\theta \qquad \tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Product-to-Sum Formulas:

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right] \qquad \cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

ALSO $a \sin x + b \cos x = k \sin(x + \alpha)$

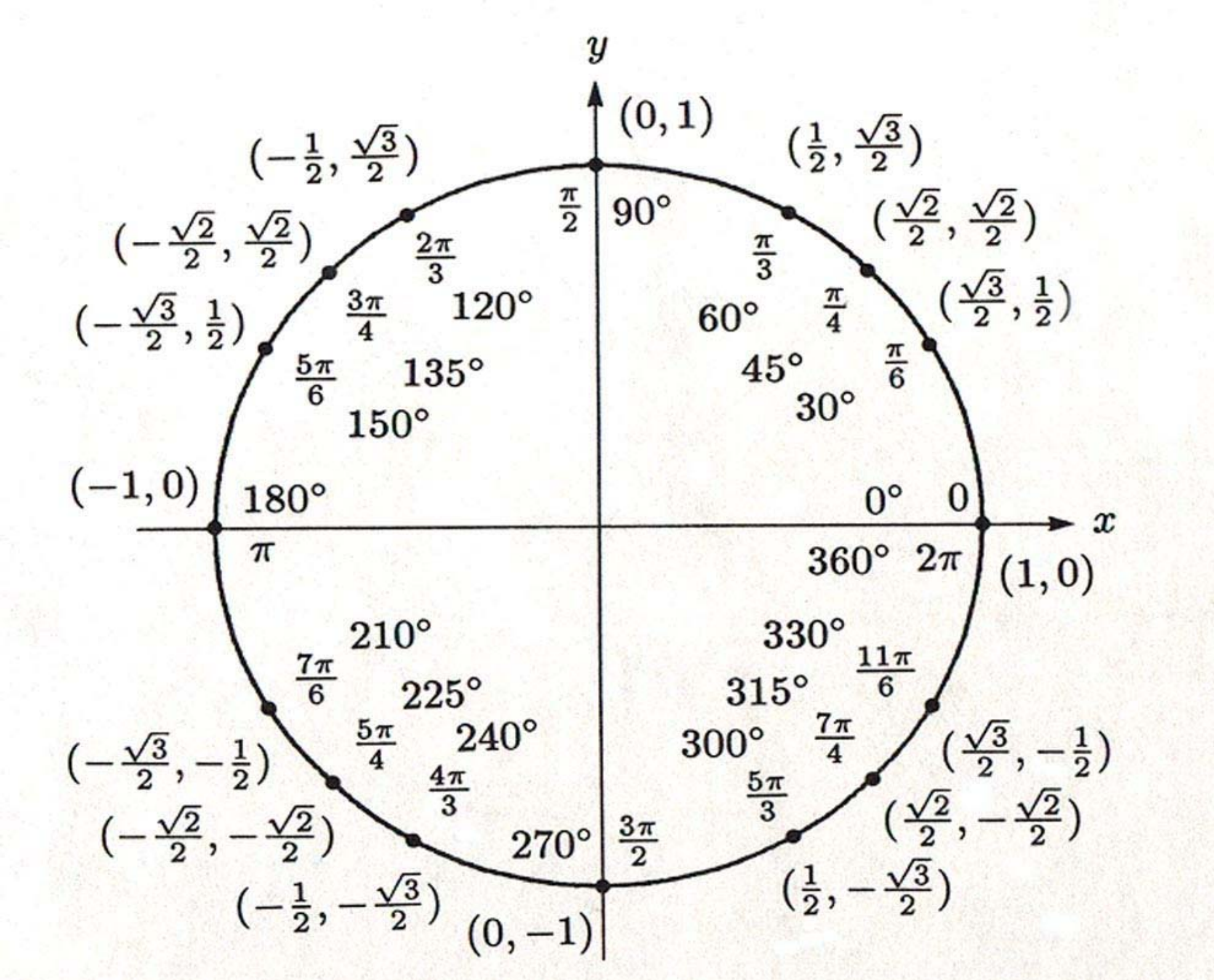
where $k = \sqrt{a^2 + b^2}$ and

$\sin\alpha = \frac{b}{k}$ and $\cos\alpha = \frac{a}{k}$

Sum-to-Product Formulas:

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

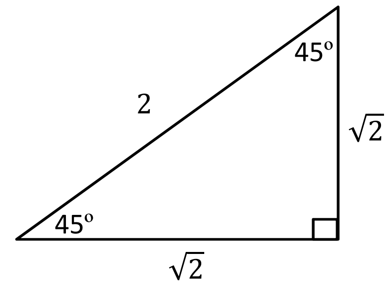


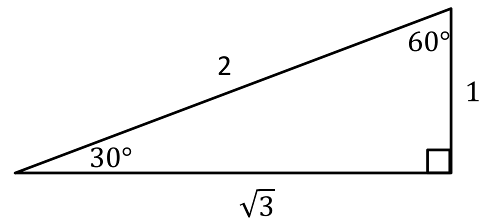
Soh Cah Toa

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotentuse}}$ $\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotentuse}}$ $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$





Degrees	Radians	Sin Θ	Cos Θ	Tan Θ	Csc Θ	Sec Θ	Cot Θ
0°	**0**	**0**	**1**	**0**	**undef**	**1**	**undef**
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$	**2**	$2\sqrt{3}/3$	$\sqrt{3}$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	**1**	$\sqrt{2}$	$\sqrt{2}$	**1**
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$2\sqrt{3}/3$	**2**	$\sqrt{3}/3$
90°	$\pi/2$	**1**	**0**	**undef**	**1**	**undef**	**0**

All **S**tudents **T**ake **C**alculus

Sine	**A**ll (positive)
Tan	**C**os

Converting Θ radians to degrees: $\Theta \cdot \dfrac{180°}{\pi}$

Converting Θ degrees to radians: $\Theta° \cdot \dfrac{\pi}{180°}$